

## Contrastive Reasoning with Inconsistent Ontologies

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**Abstract**—In this paper we present a framework for answering queries over inconsistent ontologies by using *contrastive reasoning*, the reasoning of contrasts which are expressed as contrary conjunctions like the word “but” in natural language. We argue that contrastive answers are more informative for reasoning with inconsistent ontologies, as compared with the usual simple boolean answer, i.e., either “yes” or “no”. We propose a general framework for contrastive reasoning with inconsistent ontologies. The proposed approach has been implemented in the system CRION (Contrastive Reasoning with Inconsistent ONtologies) as a reasoning plug-in in the LarKC (Large Knowledge Collider) platform. We report several experiments in which we apply the CRION system to some realistic ontologies. This evaluation shows that contrastive reasoning is a useful extension to the existing approaches of reasoning with inconsistent ontologies.

**Keywords**—contrastive reasoning; inconsistent ontology; contrastive answer; clarification formula; contrastive formula;

### I. INTRODUCTION

In real life, people frequently use the word “but” to express a contrary conjunction. For instance, one would say that “all cars are polluting, *but* hybrid cars are not polluting”; or one would say that “The conference will be held in Holland, *but* not in Amsterdam”. The first example expresses an exception that contradicts a general rule; the second example is contrary to a general expectation that one may have (namely that conferences in Holland are generally held in Amsterdam).

In the terminology of formal knowledge representation, two parts connected by “but” are *contrastive*, and reasoning with “but”-statements is called contrastive reasoning. However, contrastive reasoning has not yet captured much attention in the research of AI and logics, although there exist some previous works on this topic [1]–[3]. These previous works consider contrastive reasoning as a supplement for non-monotonic reasoning: they use non-monotonic reasoning to compute the all implications, and then determine the contrasts among them. Compared with normal boolean query answering, contrastive reasoning gives users not only an answer to the original query, but also some *contrastive answers*.

Such contrastive reasoning has two main goals:

- *Avoidance of misleading information by extending the answer.* Contrastive answers provide not only an answer to the original query, but also some relevant contrasting answers. In our introductory example, the simple answer that all cars are polluting is misleading because hybrid cars are an exception to this rule.
- *Effective influence with surprising answers.* A psychologically effective influence can be achieved by providing an additional answer that is unexpected. In our introductory Amsterdam-example, the contrastive answer that the conference is not in Amsterdam is surprising against the background expectation that all conferences in Holland will be in Amsterdam.

Contrastive reasoning exposes the contradiction that exists either between a knowledge base and external expectations (as in the conference example), or contradictions between different parts of the knowledge base (as in the polluting cars example). Because of this, contrastive reasoning is also very useful for reasoning with inconsistent ontologies, because it does not simply respond to queries with a boolean answer of either “yes” or “no”, but also provides an informative answer with some “surprising” information.

Reasoning with inconsistent ontologies is an important research topic in the Semantic Web for several reasons: i) Integration of ontologies easily leads to an inconsistency. ii) It may be ineffective or even impossible to repair inconsistencies before reasoning as the inconsistent ontologies may be too large or we may not have the right to repair inconsistencies in imported ontologies. iii) Moreover, ontologies may change at a high frequency and hence not allow for any meaningful repair. In this paper, we will therefore focus on reasoning with inconsistent ontologies.

In this paper, we propose a general framework for contrastive reasoning with inconsistent ontologies. We introduce contrastive reasoning in the general setting of First-order Logic (FOL). The proposed framework has been implemented in the system CRION as a reasoning plug-in in the LarKC platform<sup>1</sup>. We will report our experiments of applying the proposed approach to some realistic ontologies. The experiments show that contrastive reasoning is a useful and

<sup>1</sup><http://www.larkc.eu>

promising form of reasoning with inconsistent ontologies.

This paper is organized as follows: Section II presents a general framework of contrastive reasoning with inconsistent ontologies. Section III proposes and implements a method to compute contrastive answers, reports the experiments of CRION with several inconsistent ontologies. After a discussion of related work in Section IV, the last section includes conclusions and future work.

## II. FORMALIZATION OF CONTRASTIVE REASONING

The classical entailment in logics is *explosive*: any formula is a logical consequence of a contradiction. Therefore, conclusions drawn from an inconsistent knowledge base by classical inference may be completely meaningless. The general task of any system that reasons with inconsistent ontologies is: given an inconsistent ontology, return *meaningful* answers to queries. In [4], a general framework of reasoning with inconsistent ontologies has been developed. In that framework, an answer is “meaningful” if it is supported by a selected consistent subset of the inconsistent ontology, while its negation is not supported by the selected subset. PION is a system for reasoning with inconsistent ontologies, which can return such meaningful answers [4]. In the following, we will use the notation  $\models$  to denote the standard entailment, and the notation  $\approx$  to denote a nonstandard entailment.

A nonstandard entailment  $\approx$  satisfies soundness and meaningfulness. Soundness means formulas that follow from an inconsistent ontology  $\mathcal{O}$  follow from a consistent subset of the inconsistent ontology using classical reasoning. Meaningfulness of the nonstandard entailment  $\approx$  means all answers entailed are meaningful.

Properties of  $\approx$  are similar to those of the standard entailment  $\models$ . However, there is an important exception. Given an inconsistent  $\mathcal{O}$  and two formulas  $\alpha$  and  $\beta$  with  $\mathcal{O} \approx \alpha$  and  $\mathcal{O} \approx \beta$ , we can not always conclude  $\mathcal{O} \approx \alpha \wedge \beta$ . One reason for it is that the selected subset that supports  $\mathcal{O} \approx \alpha$  may differ from the selected subset that supports  $\mathcal{O} \approx \beta$ , while the union of the two subsets may be inconsistent; another reason is that  $\alpha \wedge \beta$  may be a contradiction.

Using the notion of nonstandard entailment, we can define our central notion of *contrastive answers*. Informally, a contrastive answer contains three parts:

- **Original formula.** A formula which answers the original query<sup>2</sup>;
- **Contrastive formula.** A formula which contrasts with the original answer formula;
- **Clarification formula** A formula that explains the reason why the contradiction occurs. The clarification formula need not (but may) be implied by the ontology. In some application scenarios, the clarification formulas

may be omitted in the query answer if the user does not require an explanation of contrastive answers.

This leads us to the formal definition of contrastive answers<sup>3</sup>:

*Definition 1 (Contrastive Answer):* Given an inconsistent ontology  $\mathcal{O}$ , a contrastive answer  $\mathcal{O} \approx \alpha$  **but**  $\gamma$  **although**  $\beta$  contains the following parts: an original formula  $\alpha$ , a contrastive formula  $\gamma$ , and a clarification formula  $\beta$ , such that:  $\mathcal{O} \approx \alpha$ ,  $\mathcal{O} \approx \beta$  and  $\mathcal{O} \approx \gamma$ ,  $\alpha \wedge \beta$  is not a contradiction,  $\gamma \wedge \beta$  is not a contradiction, but  $\alpha \wedge \beta \wedge \gamma$  is a contradiction.

Sometimes we leave out the clarification formula, but an answer is only contrastive if a clarification formula does exist.

*Definition 2 (Contrastive Answer without Explanation):* Given an inconsistent ontology  $\mathcal{O}$ ,  $\mathcal{O} \approx \alpha$  **but**  $\gamma$  is a contrastive answer without explanation if there exists a formula  $\beta$  such that  $\mathcal{O} \approx \alpha$  **but**  $\gamma$  **although**  $\beta$  is a contrastive answer.

The definitions above imply that contrastive answers have a nice exchange property. Namely, more contrastive answers can be obtained by exchanging the original formula, the contrastive formula and the clarification formula.

*Proposition 1 (Exchange Property of Contrastive Answers):* For an inconsistent ontology  $\mathcal{O}$  and three formulas  $\alpha$ ,  $\beta$ ,  $\gamma$ , the following hold:

- *Exchange:*  $\mathcal{O} \approx \alpha$  **but**  $\gamma$  **although**  $\beta \Rightarrow \mathcal{O} \approx \gamma$  **but**  $\alpha$  **although**  $\beta$
- *Conditional Lifting:*  $\mathcal{O} \approx \alpha$  **but**  $\gamma$  **although**  $\beta$  and  $\alpha \wedge \gamma$  is not a contradiction  $\Rightarrow \mathcal{O} \approx \beta$  **but**  $\gamma$  **although**  $\alpha$
- *Conditional Shifting:*  $\mathcal{O} \approx \alpha$  **but**  $\gamma$  **although**  $\beta$  and  $\alpha \wedge \gamma$  is not a contradiction  $\Rightarrow \mathcal{O} \approx \alpha$  **but**  $\beta$  **although**  $\gamma$

These exchange properties do not mean that  $\alpha$ ,  $\beta$  and  $\gamma$  do not differ in any way. Although they can be formally interchanged in the above way, such an interchange implies a change in the epistemological status of the formula: The original formula  $\alpha$  is the answer to the original query. Thus, it is considered to be the most important one. The contrastive formula  $\gamma$  is an additional answer. The clarification formula  $\beta$  provides some information to explain the reason why the contradiction occurs, which may be ignored if an explanation is not necessary.

Besides the exchange property above, Contrastive answers also have the expansion property: Formulas in the contrastive answer can be expanded with the conjunction  $\wedge$ .

*Proposition 2 (Expansion Property of Contrastive Answers):* For an inconsistent ontology  $\mathcal{O}$ , and three formulas  $\alpha$ ,  $\beta$ ,  $\gamma$ , the following hold:

<sup>2</sup>Note that formulas in this paper mean First-order Logic formulas. Our work is built on FOL. Without loss of generality, a Description Logic axiom can be transformed into a (conjunctive) FOL formula. Thus, in the following, we will consider only a single formula.

<sup>3</sup>In this paper, we focus on the approach of reasoning with inconsistent ontologies, in which a clarification formula is derivable from the ontology. We leave the cases of the clarification formula as an expectation (like that in the conference example) for future work.

- $\mathcal{O} \models \alpha$  **but**  $\gamma$  **although**  $\beta$  and  $\alpha \wedge \alpha' \wedge \beta$  is not a contradiction and  $\mathcal{O} \models \alpha \wedge \alpha' \Rightarrow \mathcal{O} \models \alpha \wedge \alpha'$  **but**  $\gamma$  **although**  $\beta$
- $\mathcal{O} \models \alpha$  **but**  $\gamma$  **although**  $\beta$  and  $\alpha \wedge \beta \wedge \beta'$  is not a contradiction and  $\beta \wedge \beta' \wedge \gamma$  is not a contradiction and  $\mathcal{O} \models \beta \wedge \beta' \Rightarrow \mathcal{O} \models \alpha$  **but**  $\gamma$  **although**  $\beta \wedge \beta'$
- $\mathcal{O} \models \alpha$  **but**  $\gamma$  **although**  $\beta$  and  $\beta \wedge \gamma \wedge \gamma'$  is not a contradiction and  $\mathcal{O} \models \gamma \wedge \gamma' \Rightarrow \mathcal{O} \models \alpha$  **but**  $\gamma \wedge \gamma'$  **although**  $\beta$

### III. IMPLEMENTATION AND EVALUATION

#### A. Implementation

Our approach to computing contrastive answers is an extension of PION, which uses a selection function to select a consistent subsets of the inconsistent ontology during the reasoning process. From the definition of contrastive answers, the conjunction of the original formula  $\alpha$ , the contrastive formula  $\gamma$ , and the clarification formula  $\beta$  must lead to a contradiction, i.e.,  $\{\alpha, \beta, \gamma\} \models \perp$ . That means that, given an original answer  $\alpha$  which is obtained by using the PION approach, we can try to obtain the contrastive formula and the clarification formula, by considering a *minimal inconsistent set* which contains  $\alpha$ .

The algorithm consists of the three main steps: i) extend the selected set of PION until it becomes inconsistent; ii) find a minimal inconsistent set which includes  $\alpha$ , it applies binary search to quickly find a minimal inconsistent set, which is taken from algorithm 2 in [5] with a few modifications; and iii) construct the clarification formula  $\beta$  and the contrastive formula  $\gamma$ . we pick up a clarification formula  $\beta$  in the minimal inconsistent set and construct a contrastive formula  $\gamma$  from the rest formulas. A straightforward approach to construct the formula  $\gamma$  is to take the conjunction of some subset of the minimal inconsistent set. We call that approach *Contrastive Answer by Conjunction* (CAC).

We have implemented the prototype of CRION<sup>4</sup> as a reasoning plug-in in the LarKC Platform by using Pellet<sup>5</sup> and OWLAPI<sup>6</sup>. Given a query answer in an inconsistent Description Logic (DL) ontology, CRION calculates contrastive answers based on the CAC approach. CRION uses PION<sup>7</sup> to compute the nonstandard entailment in an inconsistent ontology. Syntax-based selection function defined in [4] is used in PION.

#### B. Evaluation

We have tested the CRION prototype by applying it to inconsistent ontologies. For that test, we selected several

ontologies from the TONES ontology repository<sup>8</sup>. The profiles of the selected ontologies are shown in Table I. As the original formulas in a contrastive answer is related to a minimal inconsistent set, for each inconsistent ontology, we select the testing queries from the union of all minimal inconsistent sets calculated by using the explanation method in Pellet. We evaluate the approach of contrastive reasoning with respect to the following three aspects:

1) *Frequency*: Columns 2, 3 and 4 of Table II show that contrastive answers (CAs) occur frequently for inconsistent ontologies. For the MadCow ontology in which there is only one minimal inconsistent set, we have at least 25 contrastive answers for 5 queries. The total numbers of contrastive answers rise to hundreds (408) for the inconsistent ontologies which have dozens (51) of minimal inconsistent sets. There appear to be a reasonable number, and reasonably constant number, of contrastive answers per query across the tested ontologies (1-5).

2) *Usability*: Five researchers score the computed contrastive answers, based on the two main goals, which are discussed in Section I, namely, avoiding misleading information and improving effective influence of the answer. Those two criteria are marked based on a five point scale: 0=valueless, 1=little value, 2=some value, 3=average value, 4=high value, and 5=perfect value. The average scores are listed in the fifth column and the sixth column of Table II. For the degree of avoiding misleading information, the scores range from 3.4 (= "average value") to 4.2 (= "high value"). That means that the contrastive answers are considered to be somewhat useful to avoid misleading information for the four ontologies in our test. For the degree of improving effective influence, they have a very similar range, showing the answers to be somewhat useful for improving effective influence. The fact that all the scores in our small experiment are  $> 3$  indicates that the approach of contrastive reasoning might indeed be useful for reasoning with inconsistent ontologies.

3) *Run-time Performance*: All the experiments are carried out on an ordinary PC (with a 2.60 GHz Pentium-4 processor and 2GB of physical memory). The maximal, minimal and average computation time (in seconds) for a query by using the CAC approach are shown in columns 7, 8 and 9 of Table II. The experimental results show that for all test ontologies, the CAC computation time for computing contrastive answers for a query is limited to a small number of seconds. The maximal computation time is just a few seconds (1.1s), the minimal computation time goes even to several milliseconds (0.007s), and the average computation time is less than one second (0.26s). It shows that the calculation of contrastive answers by using the CAC approach does not significantly increase the computational

<sup>4</sup>[https://larkc.svn.sourceforge.net/svnroot/larkc/branches/Release\\_1.1\\_candidate/plugins/reason/CRION/](https://larkc.svn.sourceforge.net/svnroot/larkc/branches/Release_1.1_candidate/plugins/reason/CRION/)

<sup>5</sup><http://clarkparsia.com/pellet/>

<sup>6</sup><http://owlapi.sourceforge.net/>

<sup>7</sup><http://wasp.cs.vu.nl/sekt/pion/>

<sup>8</sup><http://owl.cs.manchester.ac.uk/repository/>, we expose their inconsistencies by adding a concept assertion for every named concept, i.e.,  $Con(the\_Con)$ .

Table I  
INFORMATION ABOUT ONTOLOGIES

Ontology	Syntax	#Cons	#Roles	#Inds	#Axioms	#MISs
MadCow	$\mathcal{ALCHQIN}(\mathbf{D})$	54	16	67	143	1
Pizza	$\mathcal{SHION}$	101	8	106	818	2
Economy	$\mathcal{ALCH}(\mathbf{D})$	338	45	818	1947	51
Transportation	$\mathcal{ALCH}(\mathbf{D})$	446	89	629	1786	62

Table II  
EXPERIMENTAL RESULTS OF CAC

Ontology	Frequency			Usability		Run-time performance		
	Num. of queries	Total num. of CAs	Avg. num. of CAs	Avg. value on AMI <sup>1</sup>	Avg. value on IEL <sup>2</sup>	Max. run time	Min. run time	Avg. run time
MadCow	5	25	5.0	4.2	4.0	0.22s	0.033s	0.12s
Pizza	8	33	4.1	3.6	3.8	1.10s	0.015s	0.26s
Economy	160	408	2.55	3.4	3.5	0.44s	0.016s	0.08s
Transportation	159	200	1.25	3.7	3.5	0.51s	0.007s	0.11s

<sup>1</sup> AMI: avoiding misleading information

<sup>2</sup> IEL: improving effective influence

cost. The CAC approach is an efficient extension to the existing reasoners with inconsistent ontologies.

#### IV. RELATED WORK

McGill and Klein address the differences in the use of covariation information implied by contrastive reasoning, which involves comparing the target episode to contrasting background instances [3]. Francez proposes the notion of bilogic as a logical treatment of a contrastive conjunction such as ‘but’, and argues that ordinary logics are not sufficient to express the contrastive nature of ‘but’, because of the neutral conjunction (‘and’) in classical logics [2]. Based on the contrastive operators proposed by Francez, a modal approach to contrastive logic is presented in [1]. Their contrastive logic is actually a simple modal logic which is an extension to the well-known S5 logic with Francez’s contrastive operator.

Default reasoning [6] is somehow similar to contrastive reasoning. It can be considered as a kind of reasoning service where the consequences may be derived only due to lacking evidence of the contrary. However, contrastive reasoning is different from default reasoning, because our approach is based on reasoning with inconsistent ontologies, whereas default reasoning is based on a non-monotonic logic.

#### V. CONCLUSIONS AND FUTURE WORK

We have presented a general framework for answering queries over inconsistent ontologies by using contrastive reasoning. It is more practical for reasoning with inconsistent ontologies, as it provides not only an original answer, but also more relevant and maybe surprising answers. We have proved that obtaining contrastive answers can be achieved by a slight extension to the existing approach for reasoning with inconsistent ontologies. Furthermore, this extension does not significantly increase the computational cost. Our proposal has been implemented in the system CRION.

We have reported several experiments with CRION and have presented an initial evaluation. The tests show that contrastive reasoning is useful and promising for reasoning with inconsistent ontologies.

This paper is the first attempt to deal with contrastive reasoning for inconsistent ontologies. Naturally, there is a lot of the future research to be done. Here are just some of them: i) In order to gain more useful answer than simple conjunction contrastive answers, other methods for obtaining contrastive answers will be very interesting. ii) Another interesting work is to express contrastive conjunctions such as “but” in the ontology language level.

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